Online Appendix to "Minimax Regret Treatment Choice with Incomplete Data and Many Treatments"

Jörg Stoye

New York University

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1 Additional Examples

The examples given in the paper illustrate cases (i) and (ii) and in both cases, yield unique δ^*. I now present another example of case (ii) and one of case (iii), both of which feature non-unique δ^*.

Example 3: An Example of Case (ii)

t	**1**	**2**	**3**	**4**	**5**
$\overline{\boldsymbol{\mu}}_t$	9	9	10	10	8
$\underline{\boldsymbol{\mu}}_t$	4	4	2	0	0
δ_t^*	$\frac{1}{5}$	$\frac{7}{20}$	$\frac{1}{4}$	$\frac{1}{5}$	0
δ_t^{**}	$\frac{7}{20}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{5}$	0
$\boldsymbol{\sigma}_t^*$	0	0	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{7}{20}$
$\boldsymbol{\mu}_t$	4	4	4	4	$\frac{14}{5}$
$\mathbf{R}(\delta^*, \mathbf{s}_t)$	≤ 5.3	≤ 5.3	5.3	5.3	5.3

The set of minimax regret treatment rules is given by the convex hull of (i.e. mixtures over) $\{\delta^*, \delta^{**}\}$. Other than that, the first six rows of the table should be self-explanatory. The last two rows serve as helpers to verify the Nash equilibria. The second last row gives the values of μ_t implied by σ^*; these are easy to verify and reveal that any δ supported on the first four treatments is a best response to σ^*. The last row gives the regret caused by state s_t given δ^*, δ^{**}, or any mixture over them. It is also easy to verify and reveals that any σ supported on $\{s_3, s_4, s_5\}$ is a best response to δ^*.

Intuitively, what can be seen in the example is that if the maximin utility treatment is not unique, then best-response conditions may fail to pin down the exact distribution of probability mass across the maximin utility treatments. This is because in this type of equilibrium, Nature does not play the states s_t corresponding to maximin treatments anyways.

Example 4: An Example of Case (iii)

t	**1**	**2**	**3**
$\overline{\boldsymbol{\mu}}_t$	2	2	2
$\underline{\boldsymbol{\mu}}_t$	1	0	0
δ_t^*	1	0	0
δ_t^{**}	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
$\boldsymbol{\sigma}_t^*$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\boldsymbol{\mu}_t$	1	1	1
$\mathbf{R}(\delta^*, \mathbf{s}_t)$	1	1	1

Again, the set of minimax regret treatment rules is given by the convex hull of $\{\delta^*, \delta^{**}\}$; obviously, this includes a nonrandomized rule. To understand this example, consider perturbing $\underline{\mu}_1$ by ε. If $\varepsilon > 0$, the decision maker would strictly prefer treatment 1, and the solution would fall into case (i) with a unique MR rule of $(1, 0, 0)$, very much like example 2 in the paper. If $\varepsilon < 0$, the decision maker would want to deviate to treatments 2 or 3, and the equilibrium would break down. The correct equilibrium then is one of case (ii), where σ^* has full support, δ^* is unique, and $\delta^* \to (1/2, 1/4, 1/4)$ as $\varepsilon \to 0$. At $\varepsilon = 0$, the solution set connects the two limits. This illustrates that case (iii) is the boundary case or switching point between (i) and (ii), which otherwise split the parameter space between them.

2 Documentation of Code

This section documents the MATLAB code snippet `MRtreat.m` to be found on my website. MRtreat.m finds all extremal δ^* as characterized in proposition 1.

Syntax. `fstar=MRtreat(bounds)`,

where `bounds` is a $(2 \cdot T)$-matrix containing upper and lower bounds as follows:

$$bounds \equiv \begin{bmatrix} \overline{\mu}_1 & \overline{\mu}_2 & \cdots & \overline{\mu}_T \\ \underline{\mu}_1 & \underline{\mu}_2 & \cdots & \underline{\mu}_T \end{bmatrix}.$$

Treatments should be arranged s.t. $\overline{\mu}_1 \geq \overline{\mu}_2 \geq \ldots \geq \overline{\mu}_T$. The output is a list of extremal minimax regret treatment rules as follows:

$$fstar \equiv \begin{bmatrix} \delta_1^{*} & \delta_2^{*} & \dots & \delta_T^{*} \\ \delta_1^{**} & \delta_2^{**} & \dots & \delta_T^{**} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_1^{*\dots*} & \delta_2^{*\dots*} & \dots & \delta_T^{*\dots*} \end{bmatrix}.$$

The set of minimax regret treatment rules is the convex hull of the matrix' rows.

The code also procides supplementary output that corresponds to the additional rows of the above tables, i.e. can be used to verify δ^*.